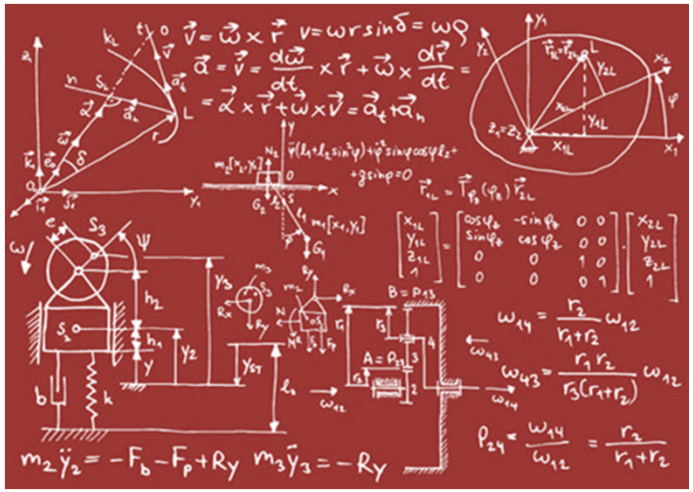
Vector Mechanics

- Scalar & Vector
- Vector Algebra
- Vector Types
- Take-Home Message



Scalar & Vector

SCALAR

What is a scalar?

A scalar is any positive or negative physical quantity that can be completely specified by its magnitude.

Examples?

length, mass, time

How do we represent a scalar?

10 meter, 50 Kilograms, 5 seconds,

Scalar & Vector

VECTORS

What is a vector?

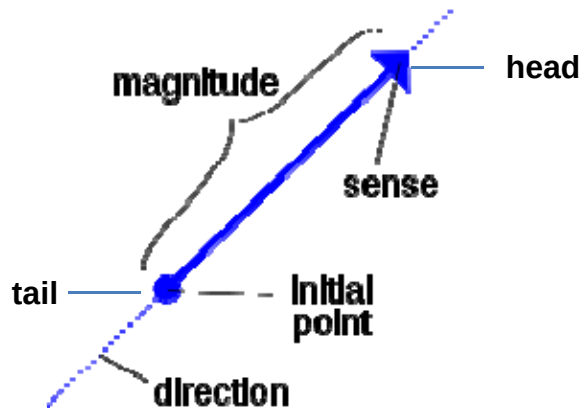
A vector is any physical quantity that requires both a magnitude and a direction for its complete description

Examples?

force, position, velocity, electric current

How do we represent vectors?

Graphically



Vector Algebra

SCALAR & VECTORS

What happens when we multiply a vector with a scalar?

Magnitude increases/decreases scalar times

Negative scalar reverses direction

Can a scalar be added to a vector?

No

How a vector is added to another vector

Triangle rule or "Head to Tail"

Parallelogram law of addition

Cartesian Vector Notation

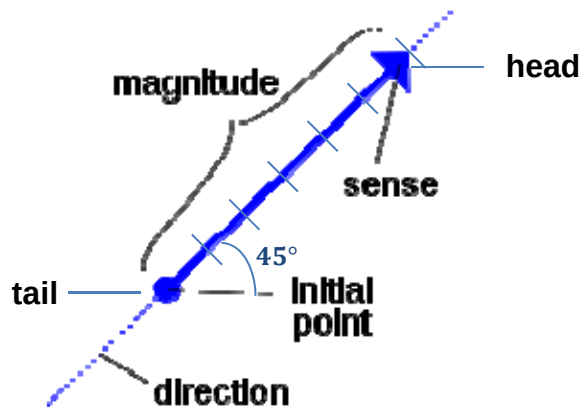
Vector Algebra

Mathematical Formulation

How do we represent vectors mathematically?

Magnitude & Direction

Let's call it vector $\vec{A}$



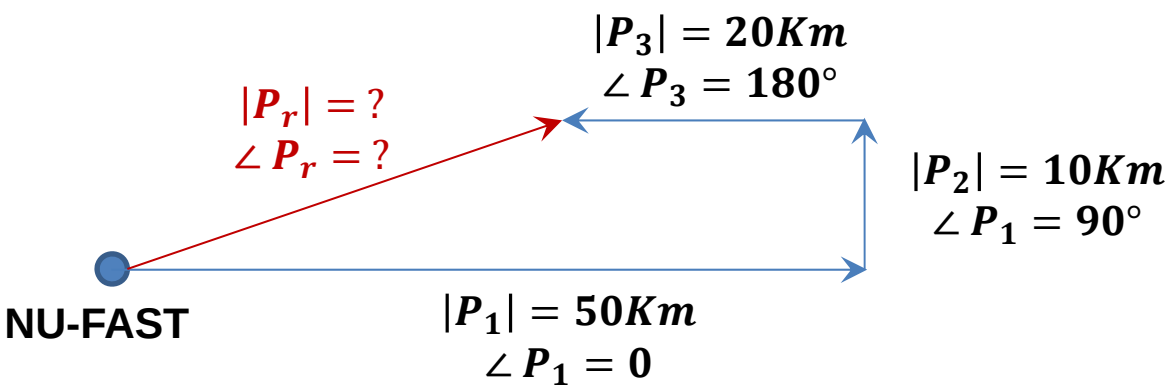
Magnitude = $|\vec{A}|$ = 6

Direction = $\angle\vec{A} = 45°$

Vector Algebra

EXAMPLE I

Suppose a student was at NU-FAST. He drives 50Km East and then he drives 10Km North followed by a 20km transit toward west.
I want to know, how far is he from campus?



Can I solve it?

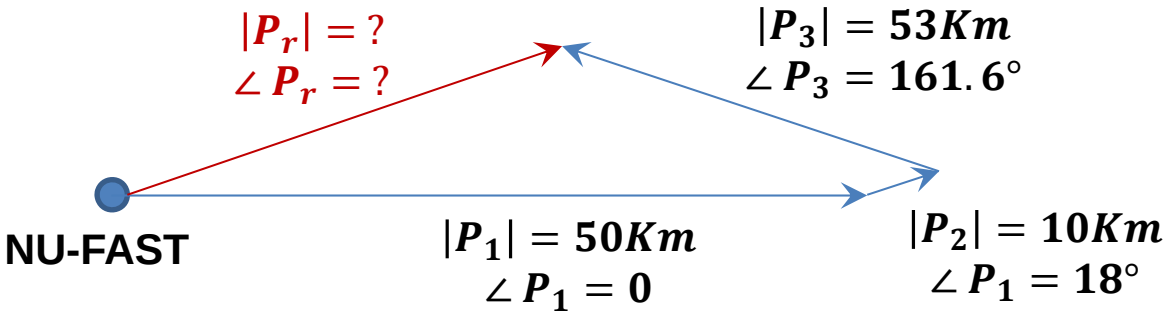
Yes! Using Head & Tail Rule + Trigonometry I can?

$$|\overrightarrow{P_r}| = \sqrt{30^2 + 10^2} = 31.623\ Km \qquad \angle\overrightarrow{P_r} = \tan^{-1}\left(\frac{10}{30}\right) = 18.635°$$

Vector Algebra

EXAMPLE I

What if we were solving the following scenario?



NOT so interesting for analytical work :(

What is the main difficulty?

Arbitrary directions

How can we solve this difficulty?

Vector Algebra

SOLUTION

Vectors Addition/Subtraction is similar to Scalar Addition/Subtraction, if they are in the same direction

Resolve vectors on to standard directions

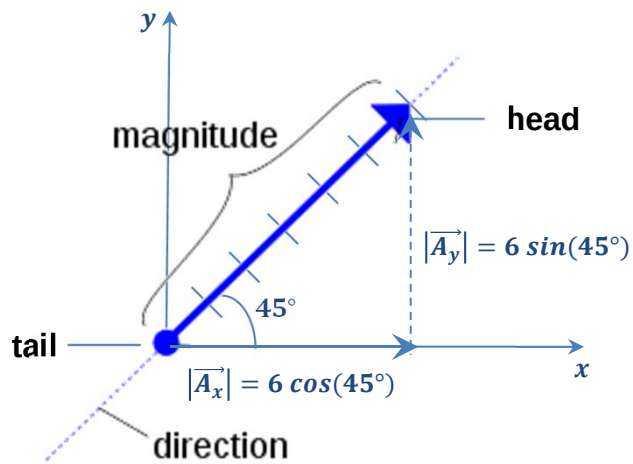
Vector Algebra

Cartesian Vector Notation

How else do we represent vectors mathematically?

Cartesian Vector Notation

Let's call it vector $\vec{A}$



$|A| = 6$

$\angle A = 45°$

$$\vec{A} = |\vec{A_x}|\,\hat{\imath} + |\vec{A_y}|\,\hat{\jmath} + |\vec{A_z}|\,\hat{k}$$

$$\vec{A} = 6\,cos\,(45°)\,\hat{\imath} + 6\,sin(45°)\,\hat{\jmath} + 0\,\hat{k}$$

$\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are unit vectors in the direction of x,y and z-axis respectively

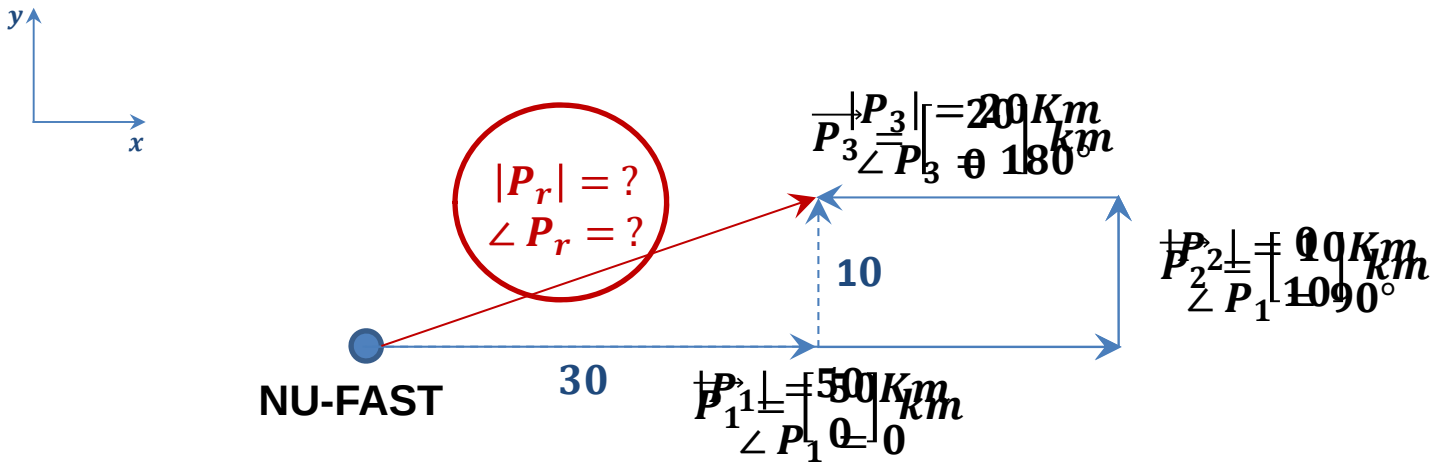
$$\vec{A} = \begin{bmatrix} |\vec{A_x}| \\ |\vec{A_y}| \\ |\vec{A_z}| \end{bmatrix} \qquad \vec{A} = \begin{bmatrix} 6\,cos\,(45°) \\ 6\,sin(45°) \\ 0 \end{bmatrix} = \begin{bmatrix} 4.243 \\ 4.243 \\ 0 \end{bmatrix}$$

Preferred writing style!

Vector Algebra

EXAMPLE I CONT.

Suppose a student was at NU-FAST. He drives 50Km East and then he drives 10Km North followed by a 20km transit toward west.
I want to know, how far is he from campus?



$$\vec{P_r} = \vec{P_1} + \vec{P_2} + \vec{P_3}$$

$$\vec{P_r} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} + \begin{bmatrix} -20 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 \\ 10 \end{bmatrix} km$$

Vector Algebra

MAGNITUDE & DIRECTION

Given rectangular components, how can we find the actual vector?

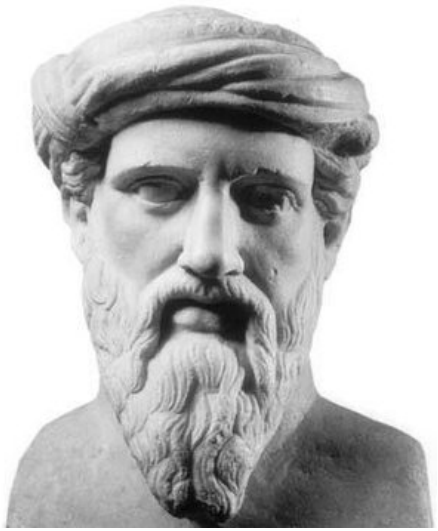
Pythagorean theorem

$$|\vec{A}| = \sqrt{|\vec{A_x}|^2 + |\vec{A_y}|^2}$$

$$\angle\vec{A} = \tan^{-1}\left(\frac{|\vec{A_y}|}{|\vec{A_x}|}\right)$$

$$|\vec{A}| = \sqrt{|\vec{A_x}|^2 + |\vec{A_y}|^2 + |\vec{A_z}|^2}$$

$$\cos\alpha = \frac{|\vec{A_x}|}{|\vec{A}|} \qquad \cos\beta = \frac{|\vec{A_y}|}{|\vec{A}|} \qquad \cos\gamma = \frac{|\vec{A_z}|}{|\vec{A}|}$$

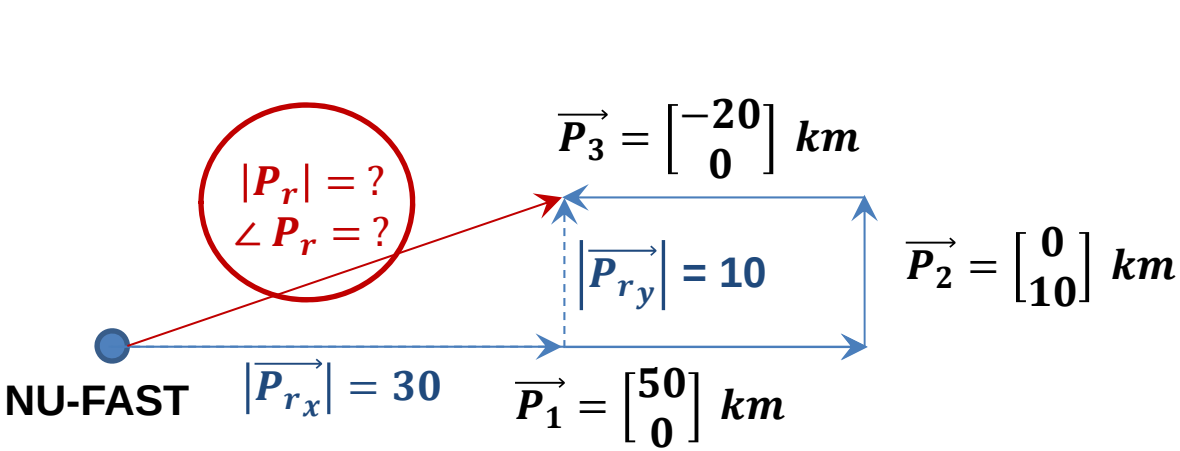


Pythagoras of Samos (570 -495 BC)

Vector Algebra

EXAMPLE I CONT.

Suppose a student was at NU-FAST. He drives 50Km East and then he drives 10Km North followed by a 20km transit toward west.
I want to know, how far is he from campus?



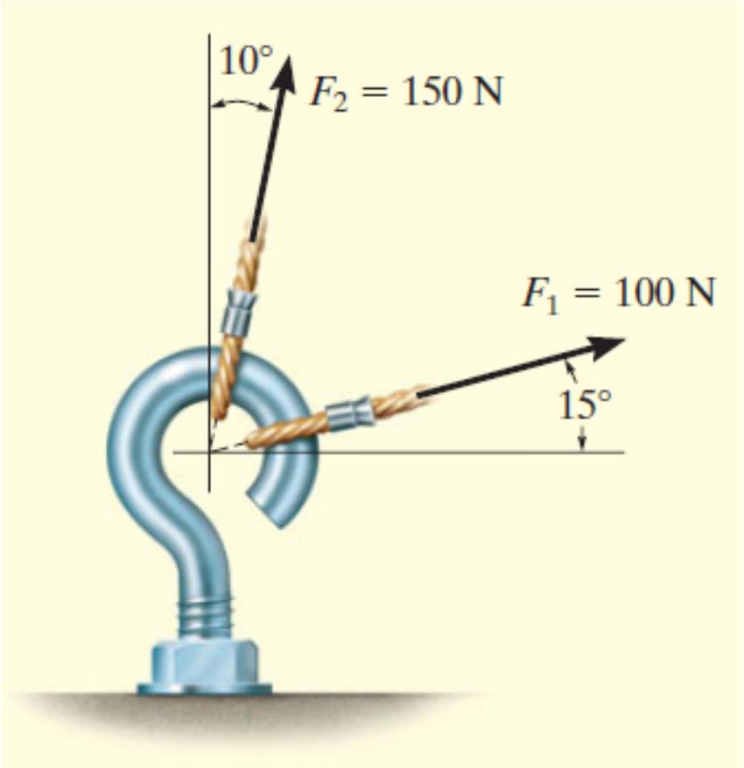
$$\overrightarrow{P_r} = \begin{bmatrix} 30 \\ 10 \end{bmatrix}$$

$$|\overrightarrow{P_r}| = \sqrt{|\overrightarrow{P_{r_x}}|^2 + |\overrightarrow{P_{r_y}}|^2} = \sqrt{30^2 + 10^2} = 31.623\ Km$$

$$\angle\overrightarrow{P_r} = \tan^{-1}\left(\frac{|\overrightarrow{P_{r_y}}|}{|\overrightarrow{P_{r_x}}|}\right) = \tan^{-1}\left(\frac{10}{30}\right) = 18.635°$$

Vector Algebra

EXAMPLE II



[1]

Determine the magnitude and direction of the resultant force.

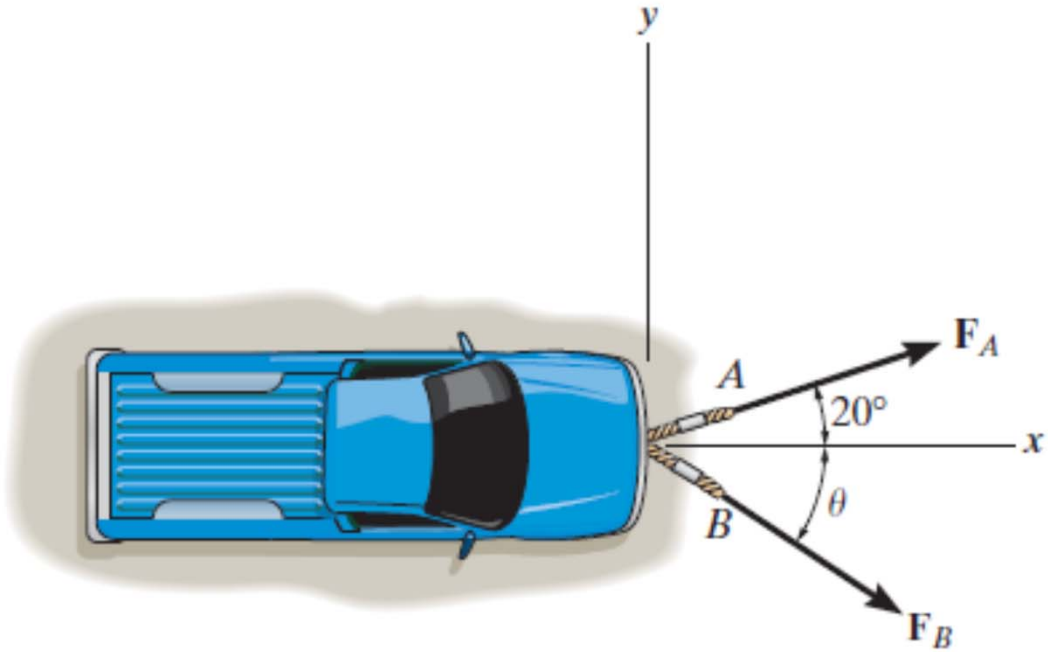
Parallelogram Law

Cartesian Vector Notation

[1] - Russell C. Hibbeler. Engineering mechanics: Statics and dynamics 12th edition, Example 2.1

Vector Algebra

EXAMPLE III



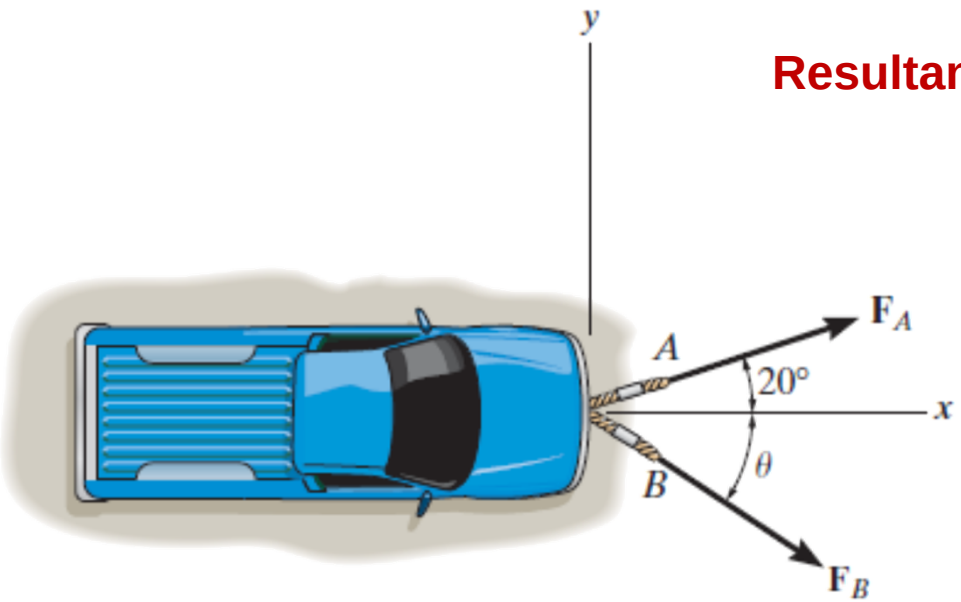
[1]

The truck is to be towed using two ropes. Determine the magnitudes of forces F_A and F_B acting on each rope in order to develop a resultant force of 950 N directed along the positive x-axis. Set θ = 50°.

[1] - Russell C. Hibbeler. Engineering mechanics: Statics and dynamics 12th edition, Problem 2.18

Resultant force of 950 N

θ = 50°.



Vector Algebra

UNIT VECTOR

What is a Unit Vector?

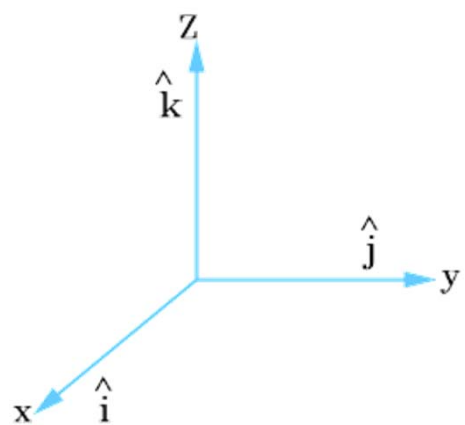
A vector of Unit Magnitude

How to find a Unit Vector along a given vector $\vec{A}$?

Dividing the given vector by its magnitude

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

A unit vector is often used to represent directions



Vector Algebra

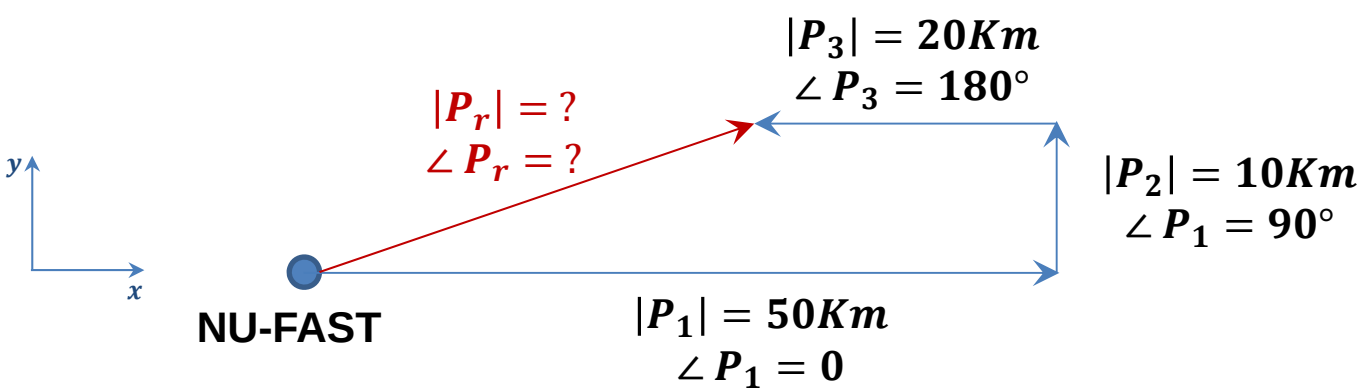
Position Vector

What is a position vector?

A position vector is defined as a fixed vector which locates a point in space relative to another point.

What do you need to define my positions?

A coordinate system



Vector Algebra

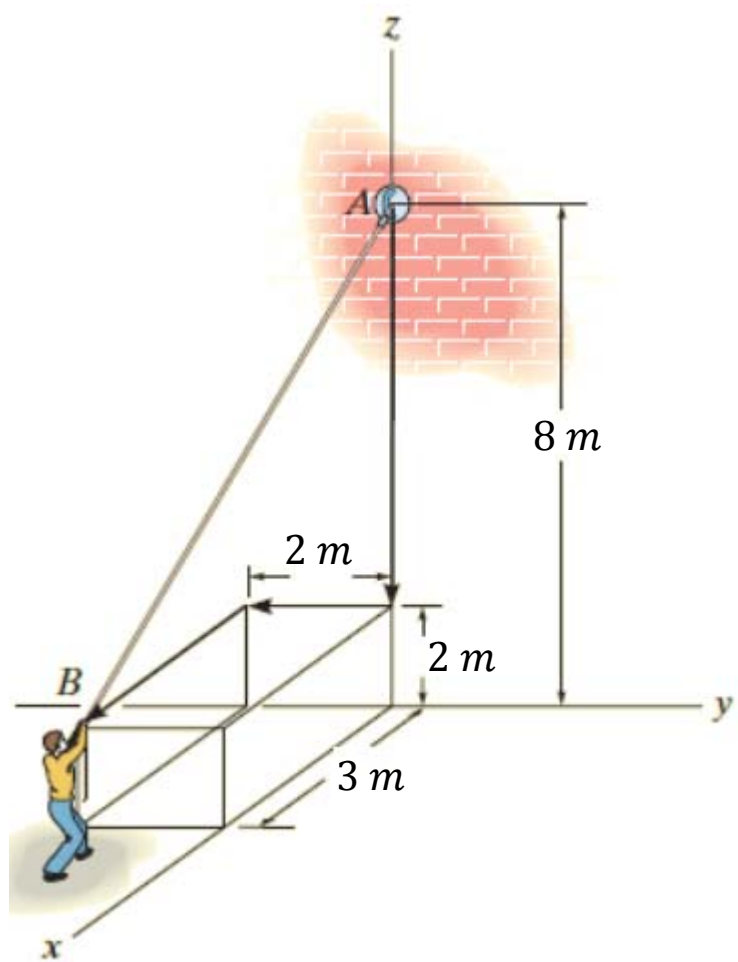
EXAMPLE IV

The man shown in the figure pulls on the cord with a force of 70N. Represent this force acting on the support A as a Cartesian vector [1]

$$\overrightarrow{AB} = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix} m$$

$$\widehat{AB} = \frac{\overrightarrow{AB}}{|AB|} = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix} \times \frac{1}{7} = \begin{bmatrix} 0.429 \\ -0.286 \\ -0.857 \end{bmatrix}$$

$$\vec{F} = 70 \times \begin{bmatrix} 0.429 \\ -0.286 \\ -0.857 \end{bmatrix} = \begin{bmatrix} 30 \\ -20 \\ -60 \end{bmatrix} N$$

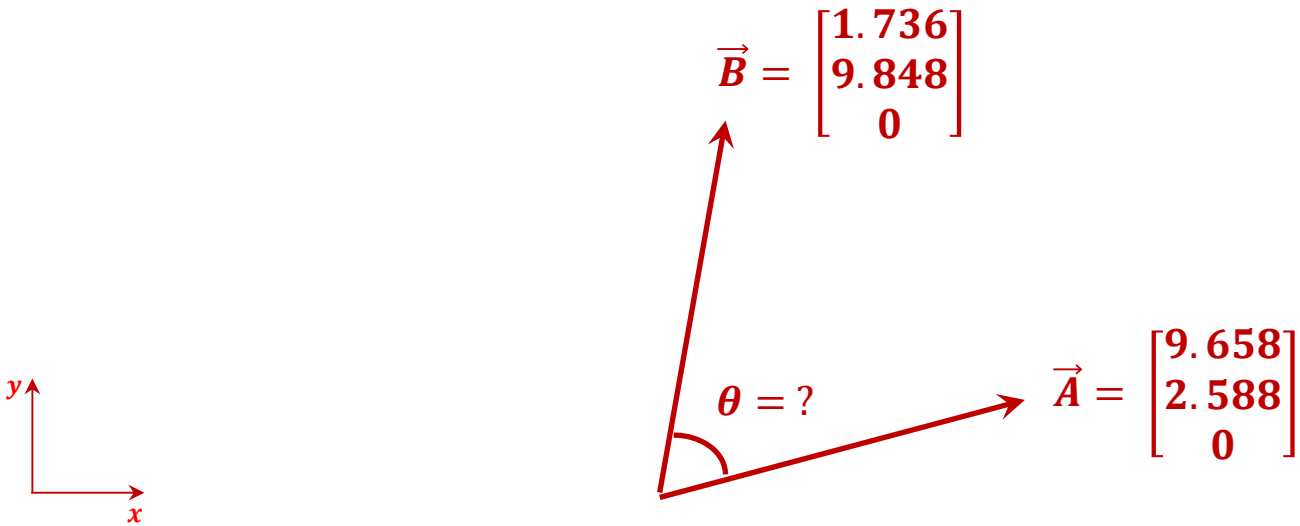


[1] - Russell C. Hibbeler. Engineering mechanics: Statics and dynamics 11th edition SI, Example 2.13, pp. 51

Vector Algebra

VECTOR PRODUCTS

Given two vectors, how can one find the angle between them?



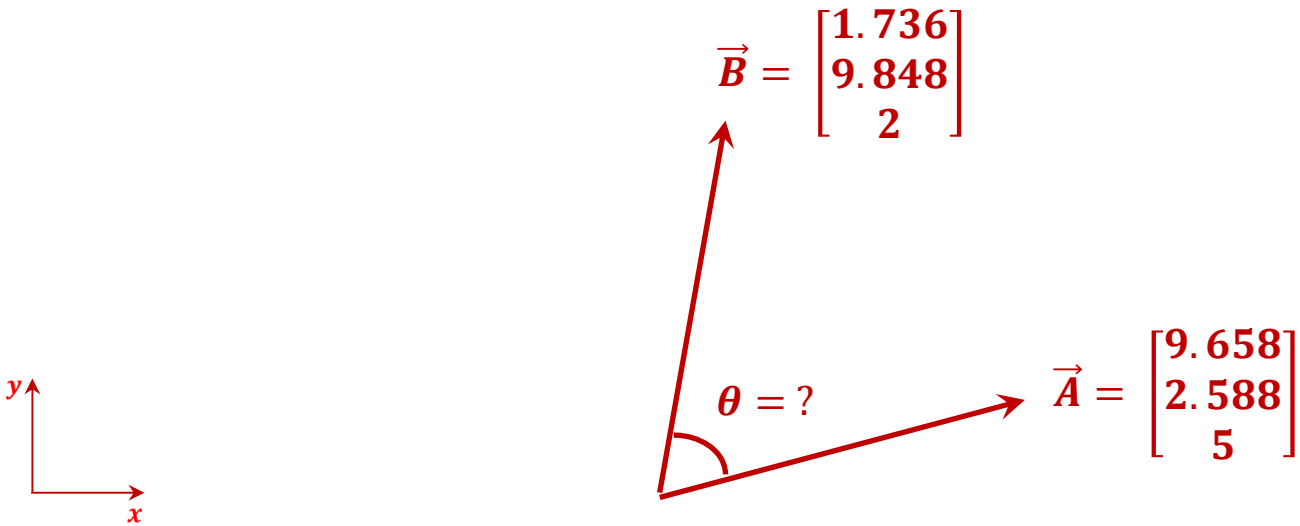
By finding angle of F_1 and F_2 because it's a 2D problem

$$\theta = 65°$$

Vector Algebra

Vector Products

Given two vectors, how can one find the angle between them?



By finding angle of F_1 and F_2 because it's a 2D problem

$$\theta = 65°$$

What if it was a 3D problem?

Dot Product :)

Vector Algebra

Vector Products

Given two vectors, how can one find the angle between them?

$$\vec{B} = \begin{bmatrix} 1.736 \\ 9.848 \\ 2 \end{bmatrix}$$

$$\vec{A} = \begin{bmatrix} 9.658 \\ 2.588 \\ 5 \end{bmatrix}$$

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|cos(\theta)$$

$$\vec{A}.\vec{B} = 52.253 \qquad |\vec{A}| = 10.198 \qquad |\vec{B}| = 11.179$$

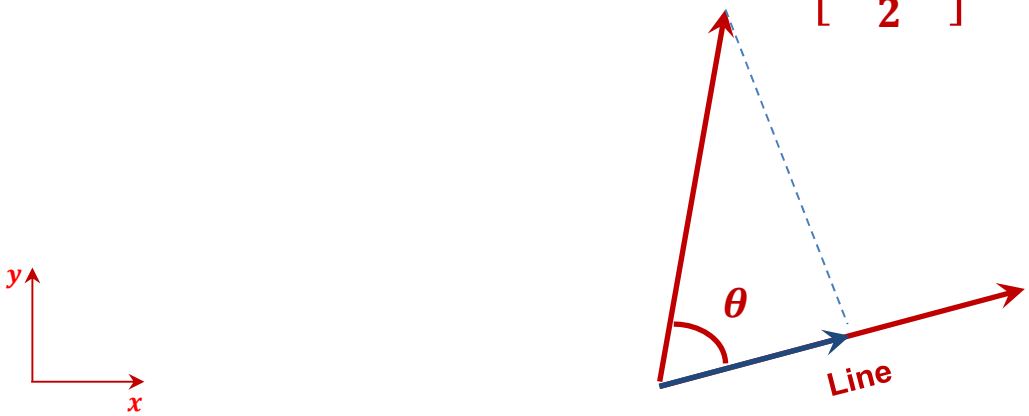
$$\theta = 62.72°$$

Vector Algebra

VECTOR PRODUCTS

How to find components of a vector parallel and perpendicular to a line?

$$\vec{B} = \begin{bmatrix} 1.736 \\ 9.848 \\ 2 \end{bmatrix}$$



$$\hat{A} = \begin{bmatrix} 0.170 \\ 0.966 \\ 0.196 \end{bmatrix}$$

Dot Product :)

How?

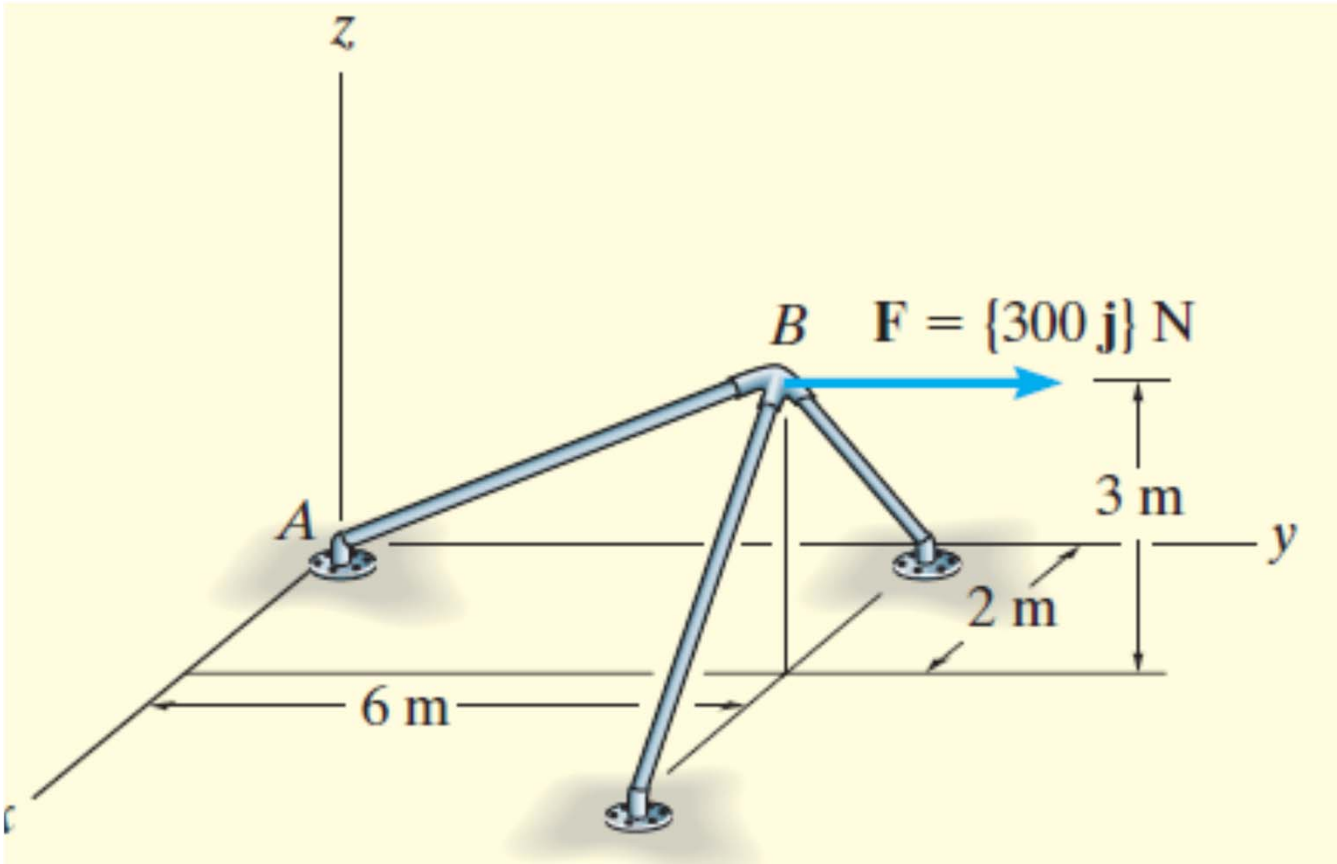
$$\vec{B}.\hat{A} = |\vec{B}||\hat{A}|cos(\theta) = |\vec{B}|\ cos(\theta)$$

Magnitude of Component of $\vec{B}$ along $\hat{A}$ = $\vec{B}.\hat{A} = 10.2$

Vector Algebra

EXAMPLE V

Determine the components of the force *F* parallel and perpendicular to member AB [1]

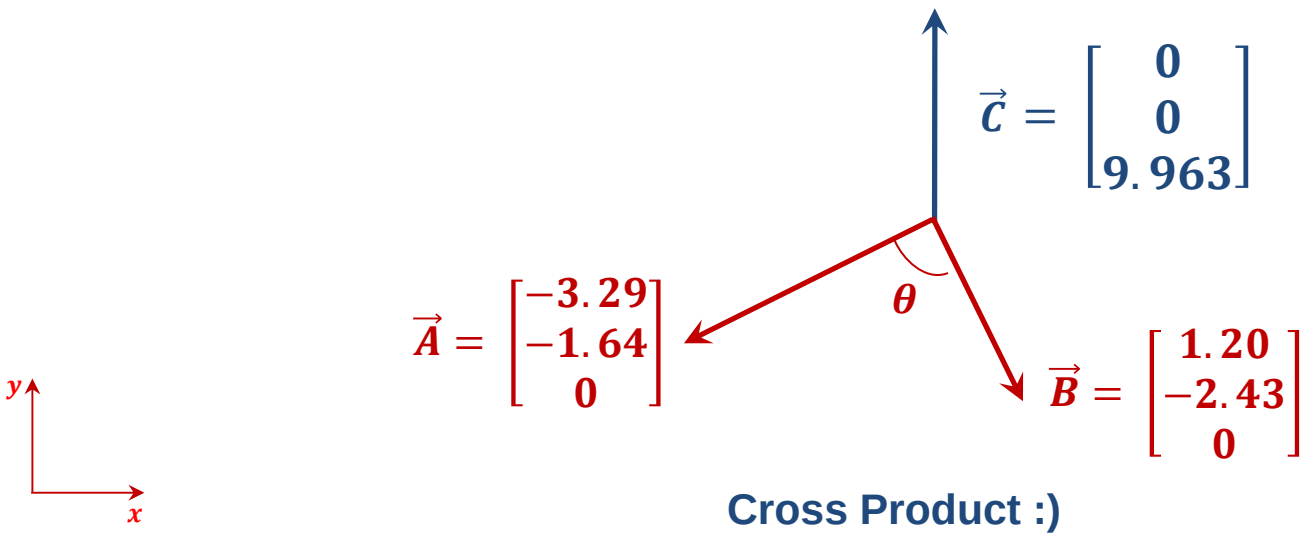


[1] - Russell C. Hibbeler. Engineering mechanics: Statics and dynamics 11th edition SI, Example 2.16, pp. 61

Vector Algebra

VECTOR PRODUCTS

How to find a vector, which is perpendicular to a plane containing two vectors?



Cross Product :)

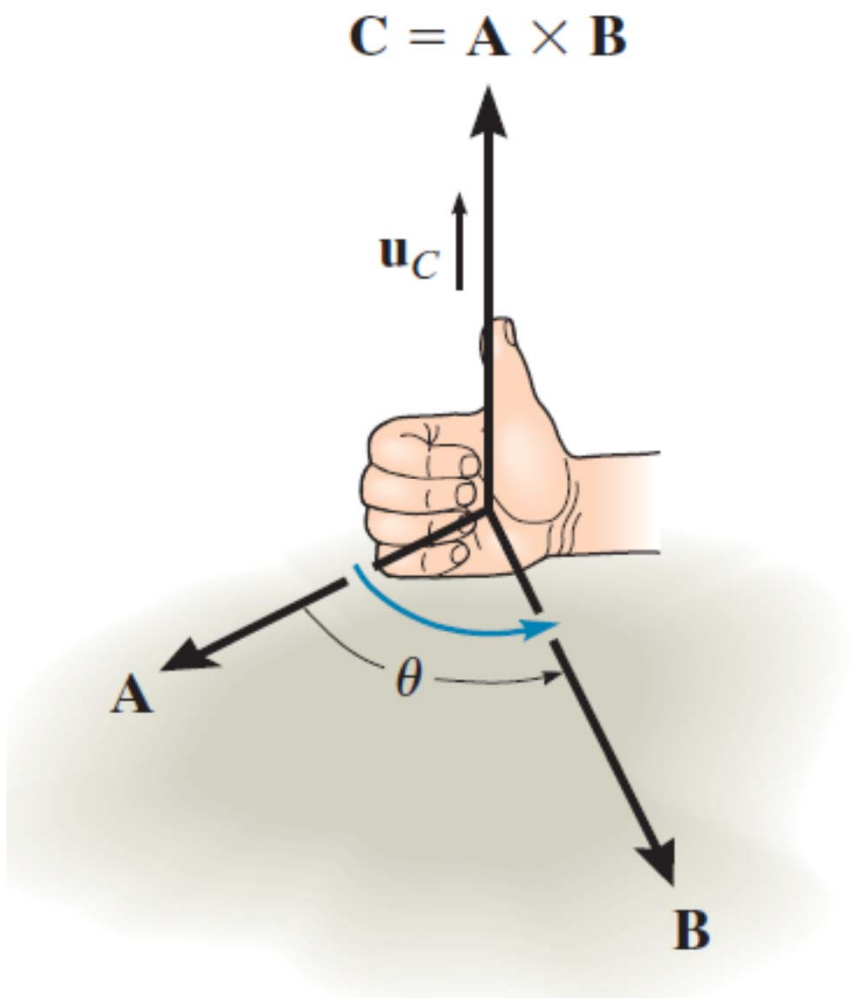
$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = |\vec{A}||\vec{B}| sin(\theta)\, \widehat{u_C}$$

$$\vec{C} = \begin{bmatrix} 0 \\ 0 \\ 9.963 \end{bmatrix}$$

Vector Algebra

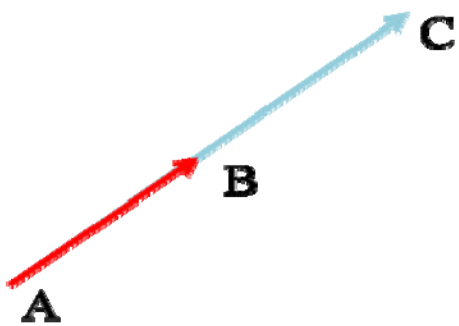
VECTOR PRODUCTS



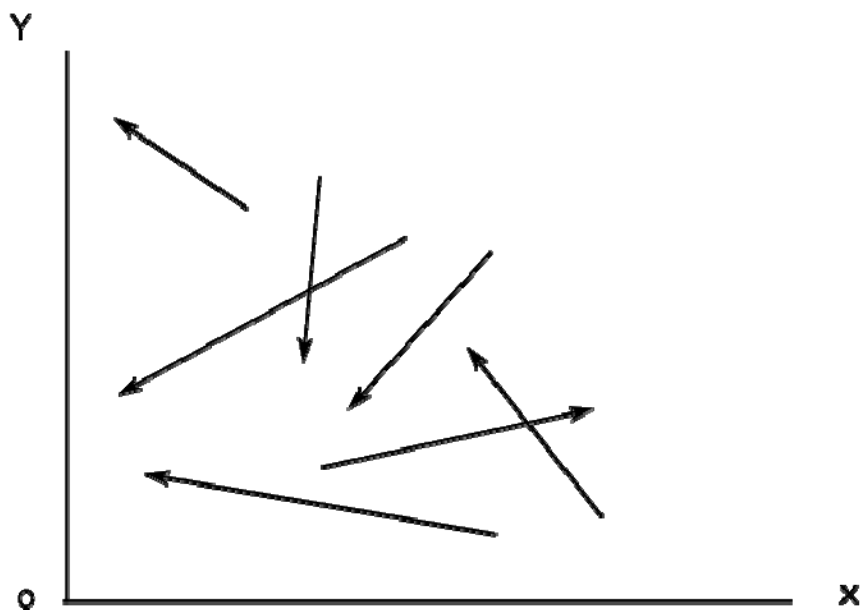
Vector Types

COLLINEAR

Collinear vectors are along the same line



Collinear

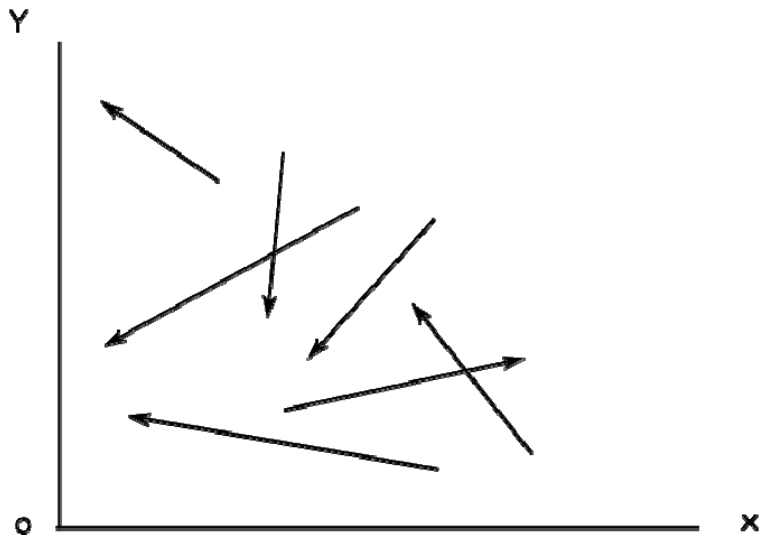


NOT Collinear

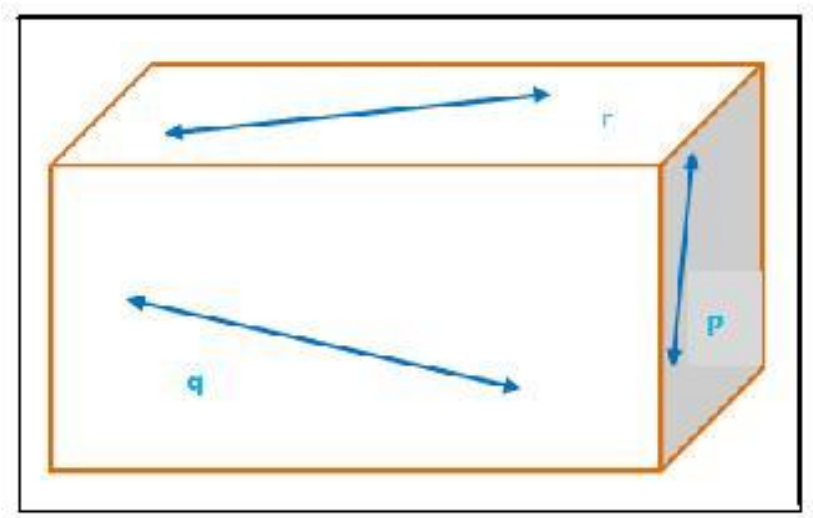
Vector Types

COPLANAR

Coplanar vectors in the same plane



Coplanar

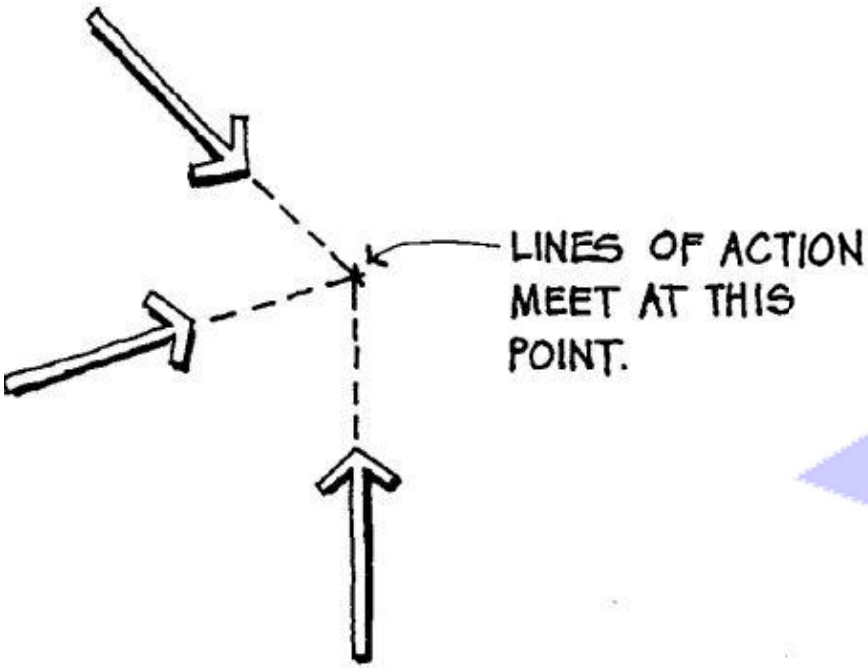


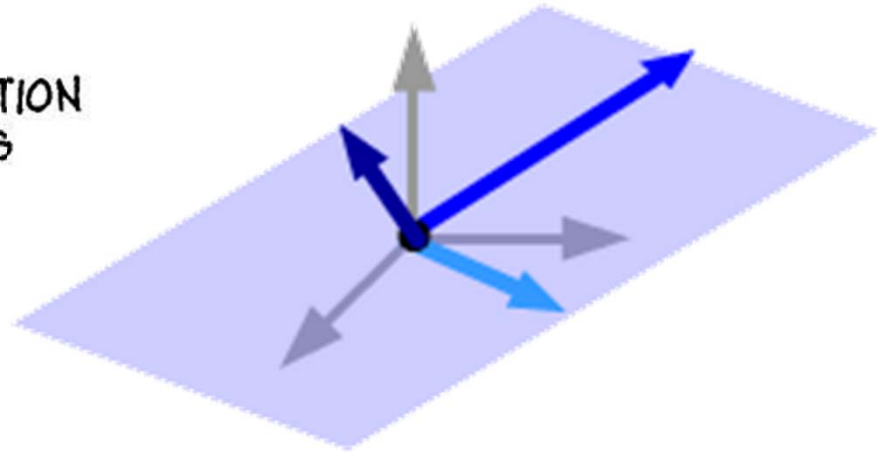
NOT Coplanar

Vector Types

CONCURRENT

Concurrent vector system is one in which the lines of action of all the vectors intersect at a common point O





Take-Home Message

PROBLEM SOLVING STEPS

1 – Mathematical Formulation

2 – Solution Strategy

3 – Actual Solution

4 – Justification & Reflection

Take-Home Message

Cartesian Vector Notation

Vectors Addition/Subtraction is similar to Scalar Addition/Subtraction, if they are in the same direction

Take-Home Message

APPLICATION OF DOT PRODUCT

1 - Finding the angle between two given vectors

2 – Finding the projection of one vector on to another